Exercise 17

Find the angle between the vectors in Exercises 9 to 11. If necessary, express your answer in terms of $\cos^{-1}$.

Solution

To obtain the angle θ between two vectors, $\mathbf{u}$ and $\mathbf{v}$, use the definition of the dot product,

$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\|\|\mathbf{v}\| \cos\theta,$$

and solve this equation for θ.

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}$$

Exercise 9

$$\mathbf{u} = -\mathbf{i} + 3\mathbf{j} + \mathbf{k},\ \mathbf{v} = -2\mathbf{i} - 3\mathbf{j} - 7\mathbf{k}$$

Use the formula for $\cos\theta$.

$$\begin{aligned}\cos\theta &= \frac{(-\mathbf{i} + 3\mathbf{j} + \mathbf{k}) \cdot (-2\mathbf{i} - 3\mathbf{j} - 7\mathbf{k})}{\sqrt{(-1)^2 + 3^2 + 1^2}\sqrt{(-2)^2 + (-3)^2 + (-7)^2}} \\ &= \frac{(-1)(-2) + (3)(-3) + (1)(-7)}{\sqrt{11}\sqrt{62}} \\ &= \frac{-14}{\sqrt{11}\sqrt{62}}\end{aligned}$$

Take the inverse cosine of both sides to get θ.

$$\theta = \cos^{-1}\left(\frac{-14}{\sqrt{11}\sqrt{62}}\right) \approx 122.4^\circ$$

Exercise 10

$$\mathbf{u} = -\mathbf{i} + 3\mathbf{k},\ \mathbf{v} = 4\mathbf{j}$$

Use the formula for $\cos\theta$.

$$\begin{aligned}\cos\theta &= \frac{(-\mathbf{i} + 3\mathbf{k}) \cdot (4\mathbf{j})}{\sqrt{(-1)^2 + 3^2}\sqrt{4^2}} \\ &= \frac{(-1)(0) + (0)(4) + (3)(0)}{4\sqrt{10}} \\ &= 0\end{aligned}$$

Take the inverse cosine of both sides to get θ.

$$\theta = \frac{\pi}{2} = 90^\circ$$

Exercise 11

$$\mathbf{u} = -\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}, \ \mathbf{v} = -\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$$

Use the formula for $\cos\theta$.

$$\begin{aligned}\cos\theta &= \frac{(-\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) \cdot (-\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})}{\sqrt{(-1)^2 + 2^2 + (-3)^2}\sqrt{(-1)^2 + (-3)^2 + 4^2}} \\ &= \frac{(-1)(-1) + (2)(-3) + (-3)(4)}{\sqrt{14}\sqrt{26}} \\ &= \frac{-17}{\sqrt{14}\sqrt{26}}\end{aligned}$$

Take the inverse cosine of both sides to get θ.

$$\theta = \cos^{-1}\left(\frac{-17}{\sqrt{14}\sqrt{26}}\right) \approx 153.0^\circ$$